Name:_____ Date:_____

Algebra Challenge: Systems & Analysis

1. Reverse Engineering

Instead of solving given equations, demonstrate your mastery by constructing equations that fit specific criteria. Work backward from the solution.

Target: x = -5 Create a **two-step equation** that results in the solution x = -5. Your equation must include: 1. A multiplication or division operationA subtraction operation
My Equation: Proof (Solve your equation to check):

2. Analyzing Structure & Order

We often use "Reverse BEDMAS" to solve two-step equations, but is that the *only* way? Let's analyze the mathematical structure.

Experiment	**Analysis**
Consider the equation: **3x + 6 = 15** **Method A:** Subtract 6 first, then divide by 3.	Does Method B work? Try it. Why is Method B generally riskier or harder to do mentally than Method A?

Method B: Divide the *entire equation* by 3 first, then subtract.	

3. Critical Analysis 🕵️

The equation below was solved incorrectly. The student made a common conceptual error regarding negative coefficients.

Equation: 5 - 2x = 17

Student Work:

1. 5 - 2x = 17-2x = 12 (Subtracted 5)x = 6 (Divided by 2)

Identify the specific error in Step 3 and explain why it happened.

4. Applying Skills: Word Problems 📝

Now that we have analyzed how to avoid common mistakes, let's put our skills to the test in real-world scenarios. Solving word problems requires two major steps: **translating** the English words into a mathematical equation, and then **solving** for the unknown variable.

💡 **Step-by-Step Approach:**

1. Identify what the question is asking for (this becomes your variable, like 'x').Look for keywords: "more than" (+), "less than" (-), "times" (×), and "total" or "is" (=).Set up your balance: [Initial Amount] [Change] = [Final Result].

The Pizza Party Problem

Marcus and his 3 friends (4 people total) shared a large pizza. They also bought a bottle of soda for $4.50. If their total bill was $26.50, how much did each person pay for their share of the pizza?

Assume the soda cost was added to the total after the pizza was split.

Define your variable (x):

Write your equation:

Final Answer (x = ?):

1. **Which equation represents this scenario?** Sarah started with some money in her savings account. She withdrew $20 each week for 4 weeks. Now she has $120 left. Let 'm' be her starting amount.	a) m + 80 = 120 b) m - 20 = 120 c) m - 80 = 120 d) 4m - 20 = 120

The Smartphone Plan:

A cell phone company charges a flat monthly fee of $35 plus $0.10 for every text message sent. If Chloe's bill last month was $42.50, how many text messages (t) did she send? Show all your work.

Equation:

Steps to solve:

5. Mathematical Modeling

Real-world algebra isn't just about solving—it's about translation. Translate these scenarios into models.

The Constraint Problem:

A rectangle has a perimeter of 50 meters. The length is 5 meters less than twice the width.
Define your variable, write the equation, and determining the dimensions.

Variable defined:

Equation:

Solution:

The Inverse Task:
Create a realistic word problem that would be solved using the equation:
150 - 15x = 30

6. Abstract Generalization 🧠

Algebra is powerful because it works for *any* number. Look at the standard linear equation form below.

Solve the literal equation **y = mx + b** for **x**. (Isolate x in terms of y, m, and b).

Answer Key

1. Reverse Engineering

Answers will vary. Example: 3x - 5 = -20. (3(-5) - 5 = -15 - 5 = -20).

2. Analyzing Structure & Order

Method B works (x + 2 = 5, so x = 3). It is riskier because students often forget to divide *every* term (like the 6) by 3, or it creates messy fractions if the numbers aren't compatible.

3. Critical Analysis 🕵️

The student divided by positive 2 instead of negative 2. In the term '-2x', the negative sign is attached to the coefficient. The correct step is x = 12 / -2, so x = -6.

4. Applying Skills: Word Problems 📝

Answer:
Variable x = cost per person for pizza. Equation: 4x + 4.50 = 26.50. Solve: 4x = 22.00, so x = $5.50 per person.

Multiple Choice:
1. m - 80 = 120. (Since 4 weeks × $20 = $80 total withdrawn).

Equation: 35 + 0.10t = 42.50.
Step 1: Subtract 35 from both sides: 0.10t = 7.50.
Step 2: Divide by 0.10: t = 75 text messages.

5. Mathematical Modeling

Let w = width. Length = 2w - 5. Equation: 2(w) + 2(2w - 5) = 50. Solution: w = 10, l = 15.

Example: You have $150. You spend $15 per week on snacks. After how many weeks will you have $30 left?

6. Abstract Generalization 🧠

x = (y - b) / m