



Name: \_\_\_\_\_

Date: \_\_\_\_\_

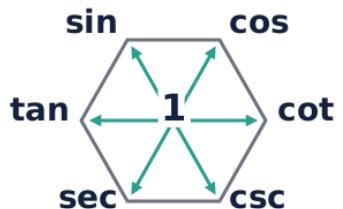
## Trigonometric Identities:

### Fundamental Recall

Reviewing the Pythagorean and Reciprocal identities is essential before tackling complex proofs. Complete the identities below to warm up your memory.

**Note:** Use plain text notation (e.g.,  $\sin^2 x$  for sine squared).

**Word bank:** 1,  $\cos^2 \theta$ ,  $\csc^2 \theta$ ,  $\sec^2 \theta$ ,  $\tan^2 \theta$ ,  $\sec \theta$ ,  $\csc \theta$ ,  $\cot \theta$



1. The primary Pythagorean Identity states that  $\sin^2 \theta + \underline{\hspace{2cm}} = 1$ .
2. Rearranging the identity  $1 + \tan^2 \theta = \underline{\hspace{2cm}}$  allows us to substitute secants for tangents.
3. The reciprocal of  $\sin \theta$  is  $\underline{\hspace{2cm}}$ .
4. In terms of quotients,  $\cos \theta \div \sin \theta$  is equivalent to  $\underline{\hspace{2cm}}$ .

### Simplifying Expressions

Simplify the following expressions to a single trigonometric term or a number. Do not skip steps mentally—ensure you can justify the simplification.

1. Simplify the expression:  $(1 - \cos^2 x) \div \sin x$

- a)  $\cos x$
- b)  $\sin x$
- c)  $\tan x$

	d) $\csc x$
2. Which expression is equivalent to: $\tan x$ $\cos x$	a) $\sin x$ b) $\cos^2 x$ c) 1 d) $\sec x$
3. Simplify: $(\sec x \div \csc x)$	a) $\sin x$ b) $\cot x$ c) $\tan x$ d) 1

## Double Angle & Compound Formulas



**Strategy Tip:** When dealing with  $\cos 2x$ , you have three options:  $\cos^2 x - \sin^2 x$ ,  $2\cos^2 x - 1$ , or  $1 - 2\sin^2 x$ . Always look at the *other* terms in the expression to decide which version will help you eliminate terms.

Use the double angle formulas to expand and simplify the expressions below. Show your work in the right-hand column.

Expression	Simplify (Show Steps)
$2 \sin(3x) \cos(3x)$	
$(\cos 2x + 1) \div 2$	

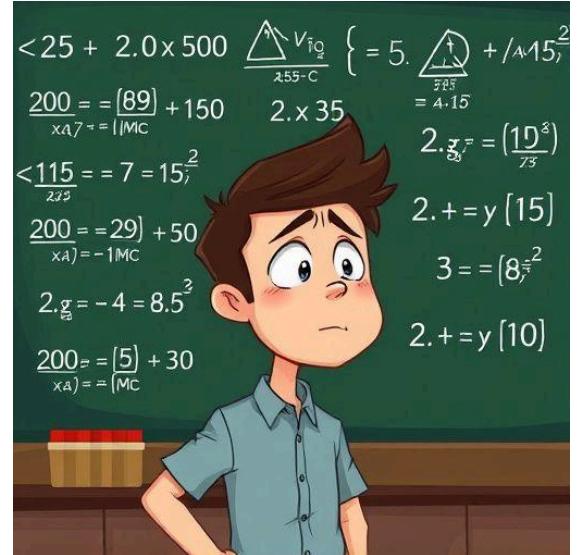
## Proof & Analysis

In this section, you will analyze a flawed proof and construct your own proof. Remember, when proving identities, work on one side only (usually the more complex side) until it matches the other.

**Error Analysis:** A student attempted to prove that  $(\sin x + \cos x)^2 = 1 + \sin 2x$ . Identify the specific algebraic error they made or explain why their first step was insufficient.

*Student's work:*

" $(\sin x + \cos x)^2 = \sin^2 x + \cos^2 x = 1$ . Therefore,  $1 = 1 + \sin 2x$ , which is false."



## Prove the Identity

**Prove that:**

$$(1 - \cos 2x) \div \sin 2x = \tan x$$

Show all steps clearly, starting with the Left Hand Side (LHS).

# Answer Key

## Fundamental Recall

### Gap Fills:

1. The primary Pythagorean Identity states that  $\sin^2\theta + \cos^2\theta = 1$ .
2. Rearranging the identity  $1 + \tan^2\theta = \sec^2\theta$  allows us to substitute secants for tangents.
3. The reciprocal of  $\sin \theta$  is  $\csc \theta$ .
4. In terms of quotients,  $\cos \theta \div \sin \theta$  is equivalent to  $\cot \theta$ .

## Simplifying Expressions

### Multiple Choice:

1.  $\sin x$
2.  $\sin x$
3.  $\tan x$

## Double Angle & Compound Formulas

Row 1:  $\sin(2x)$  (Double angle sine formula). Row 2:  $\cos^2 x$  (Rearranged double angle cosine formula).

## Proof & Analysis

The student incorrectly expanded the binomial.  $(\sin x + \cos x)^2$  expands to  $\sin^2 x + 2\sin x \cos x + \cos^2 x$ , NOT just  $\sin^2 x + \cos^2 x$ . They missed the middle term, which equals  $\sin 2x$ .

$$\begin{aligned} \text{LHS} &= (1 - (1 - 2\sin^2 x)) / (2\sin x \cos x) \\ &= (2\sin^2 x) / (2\sin x \cos x) \\ &= \sin x / \cos x \\ &= \tan x \\ &= \text{RHS} \end{aligned}$$